Two-Beam Spin Noise Spectroscopy

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We propose a method of two-beam spin noise spectroscopy to test the spin transport at equilibrium via analysis of correlations between time-shifted spin fluctuations at different space locations. This method allows one to determine the strength of spin-orbit interaction and spin relaxation time and separate spin noise of conducting electrons from the background noise of localized electrons. We formulate a theory of two-beam spin noise spectroscopy in semiconductor wires with Bychkov-Rashba spin-orbit interaction taking into account several possible spin relaxation channels and finite size of laser beams. Our theory predicts a peak shift with respect to the Larmor frequency to higher or lower frequencies depending on the strength of spin orbit interaction and distance between the beams. The two-beam spin noise spectroscopy could find applications in experimental studies of semiconductors, emergent materials and many other systems.

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The ability to understand and control the electron spin polarization in semiconductors will have profound technological impacts^{1,2}. For this purpose, it is important to obtain information about interactions that spins of conducting electrons experience in semiconductors as these interactions can play a significant role in the processes of spin relaxation and control. Experimentally, the electron spin polarization and its fluctuations (spin noise) in semiconductors are often detected using the optical Faraday or Kerr rotation technique (see, e.g., Refs. 3–5). The Faraday rotation technique relies on the fact that a linearly polarized light propagating through a material with a net magnetic moment is subject to a rotation of its plane of polarization. The amount of rotation induced is proportional to the internal local magnetization of material and thus can provide information about electron spin polarization and its fluctuations.

The major advantage of spin noise spectroscopy^{4–8} to all other probes of semiconductor spin dynamics lies (e.g., optical pump-probe measurements⁹) in the fact that in principle no energy has to be dissipated in the sample, i.e., it exclusively yields the intrinsic, undisturbed spin dynamics⁵. In addition, the spin noise spectroscopy allows accumulation of a large statistics that smoothes out the statistical noise in the data. At the present time, experimentalists utilize a single beam setup measuring a noise correlation function at the same space location^{4–7}.

We suggest a two-beam spin noise spectroscopy measuring both time and space correlations of spin fluctuations. Fig. 1 presents a possible experimental setup involving two laser beams (that may or may not overlap) separated by a distance d. This setup captures the propagation of

spin fluctuations between different space locations providing a unique information regarding spin transport at equilibrium. To the best of our knowledge, there are no other available experimental approaches to obtain such information.

In our theory, we assume that at the initial moment of time t=0 (that can be arbitrarily selected) the vector of spin polarization density is given by a vector of continuous random variables $\mathbf{S}(\mathbf{r},0) = \boldsymbol{\xi}(\mathbf{r})$ such that $\langle \xi_i(\mathbf{r}) \rangle = 0$ and $\langle \xi_i(\mathbf{r})\xi_j(\mathbf{r}') \rangle = \lambda \delta(\mathbf{r} - \mathbf{r}')\delta_{ij}$, where $\langle ... \rangle$ denotes averaging over different realizations, i,j=x,y,z, and λ is a parameter describing the strength of spin fluctuations. Using statistical considerations¹⁰, one can find that $\lambda = n/4$, where n is 2D electron density. For a given realization of initial spin polarization density, the spin polarization at t>0 can be written using a Green's

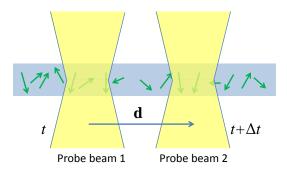


FIG. 1. Two-beam spin noise spectroscopy. Correlations between time-shifted intrinsic spin fluctuations at different space locations provide information about spin transport at equilibrium. Here, \mathbf{d} is the displacement of Beam 2 with respect to Beam 1.

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function G_{ij} of spin diffusion equation

$$S_i(\mathbf{r},t) = \int_A G_{ij}(\mathbf{r},t;\mathbf{r}',0)S_j(\mathbf{r}',0)d\mathbf{r}' + \chi_i(\mathbf{r},t), \quad (1)$$

where A is the sample area, $\chi_i(\mathbf{r},t)$ is a stochastic function describing the spin fluctuations created in the system at t > 0 so that $\chi_i(\mathbf{r},0) = 0$, and $d\mathbf{r}' = dx'dy'$ (in 2D case). Next, we note that each beam in the optical setup shown in Fig. 1 averages the space distribution of the Faraday rotation angle $\theta(\mathbf{r},t) = \kappa S_z(\mathbf{r},t)$ according to

$$\bar{\theta}_m(t) = \frac{1}{P_0} \int_A I_m(\mathbf{r}) \theta(\mathbf{r}, t) d\mathbf{r} = \frac{\kappa}{P_0} \int_A I_m(\mathbf{r}) S_z(\mathbf{r}, t) d\mathbf{r},$$
(2)

 κ is a constant that couples z-component of spin polarization density with a local value of Faraday rotation angle, P_0 is the integrated laser beam intensity (power), and $I_m(\mathbf{r})$ is the space distribution of m-th beam intensity, where m = 1, 2.

In the case of two-beam spin noise spectroscopy (we assume that two beams with the same incident polarization and intensity are shifted by \mathbf{d} as shown in Fig. 1), the experimentally determined correlation function is

$$R(t) = \langle \bar{\theta}_1(0)\bar{\theta}_2(t)\rangle. \tag{3}$$

Using Eqs. (1)-(3), independence of $\chi(\mathbf{r}, t)$ from the initial spin density distribution $\xi(\mathbf{r})$, and the expression $\langle \xi_i(\mathbf{r})\xi_j(\mathbf{r}')\rangle = \lambda\delta(\mathbf{r} - \mathbf{r}')\delta_{ij}$ (introduced above Eq. (1)), we find the general equation determining the second order spin noise correlation function:

$$R(t) = \frac{\lambda \kappa^2}{P_0^2} \int_A \int_A I(\mathbf{r}) I(\mathbf{r}' - \mathbf{d}) G_{zz}(\mathbf{r}, t; \mathbf{r}', 0) d\mathbf{r} d\mathbf{r}'.$$
(4)

The Fourier transform of R(t) with respect to t is the noise power spectrum

$$S(\omega) = \int_{0}^{\infty} R(t) \cos(\omega t) dt.$$
 (5)

We emphasize that although Eq. (4) contains only zz component of the Green function, the latter (as a solution of a system of spin diffusion equations) incorporates both transverse and longitudinal dynamics of spin polarization

Eqs. (4) and (5) are quite general and can be applied to many different experimental situations. Here, we consider a specific experimental system – a wire with the Bychkov-Rashba¹¹ spin-orbit interaction (SOI) and further develop the theory of two-beam spin noise spectroscopy for this particular case. We assume that the wire is placed in an in-plane magnetic field perpendicular to the wire axis (to shift a peak in the noise power spectrum from zero frequency) and, in addition to the D'yakonov-Perel' (DP) spin relaxation channel, there are

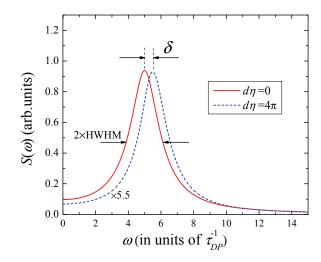


FIG. 2. Spin noise power spectrum in one-dimensional wires for different values of the beams' separation d. The direction of the peak's displacement is determined by the direction of the displacement of the second beam with respect to the first one along the wire. These curves were obtained using the parameter values $R_0 \eta = 1.5\pi$, $\gamma = 0.2/\tau_{DP}$, and $\omega_L = 5/\tau_{DP}$.

additional spin relaxation mechanisms such as the Elliot-Yaffet mechanism^{12,13} and/or electron spin relaxation on nuclear spins¹⁴. Assuming a Gaussian distribution of the incident laser beam intensities along the x-direction, namely, $I(x) \propto \exp(-x^2/(2R_0^2))$, where R_0 is the beam radius, and using the Green's function of one-dimensional spin diffusion equation,

$$G_{zz}(x,t;x',0) = \frac{1}{\sqrt{4\pi Dt}} e^{-\frac{(x-x')^2}{4Dt} - \gamma t} \cos(\eta(x-x') - \omega_L t),$$
(6)

where D is the diffusion coefficient, η is the spin precession angle per unit length (due to the Bychkov-Rashba SOI), γ is the spin relaxation rate due to additional to the DP spin relaxation channels, and ω_L is the Larmor frequency, we find (with a help of Eq. (4)) the noise correlation function

$$R(t) \propto \frac{\cos\left(\omega_L t + \frac{d\eta D t}{D t + R_0^2}\right)}{\sqrt{D t + R_0^2}} e^{-\frac{R_0^2 \eta^2 D t + d^2/4}{D t + R_0^2} - \gamma t}.$$
 (7)

Unfortunately, the Fourier transform of Eq. (7) can not be explicitly obtained. It is possible, however, to derive a closed analytical expression in a limiting case when the short times provide the main contribution to the Fourier transform of Eq. (7). Strictly speaking, this limiting case is realized if $R_0|\eta|\gg 1$ and $d/(R_0^2|\eta|)\ll 1$. Neglecting Dt compared to R_0^2 in Eq. (7), we rewrite this equation as

$$R(t) \propto \cos\left[\left(\omega_L + \frac{d\eta D}{R_0^2}\right)t\right] e^{-(\gamma + \eta^2 D)t} e^{-\frac{d^2}{4R_0^2}}.$$
 (8)

The Fourier transform of Eq. (8) gives the spin noise

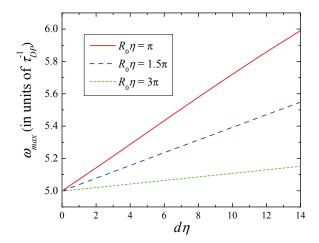


FIG. 3. Position of the peak in the spin noise power spectrum (Fig. 2) as a function of beams' separation d. These curves were obtained using the parameter values $\gamma = 0.2/\tau_{DP}$ and $\omega_L = 5/\tau_{DP}$.

power spectrum

$$S(\omega) \propto \frac{\tau_s}{1 + \tau_s^2 (\omega_L + \delta - \omega)^2} e^{-\frac{d^2}{4R_0^2}},\tag{9}$$

where $\tau_s^{-1} = \gamma + \tau_{DP}^{-1}$ is the total spin relaxation rate, $\tau_{DP} = (\eta^2 D)^{-1}$ is the D'yakonov-Perel' spin relaxation time^{15,16}, and $\delta = d\eta D/(R_0^2)$ is the peak shift from its position at d = 0. According to Eq. (9), the spin noise spectrum shows a Lorentzian peak centered at $\omega' = \omega_L + \delta$; the resonance width is determined by the combined (due to all possible spin relaxation processes) spin relaxation time τ_s . Importantly, the resonance is shifted from ω_L by $\delta = d\eta D/R_0^2$ depending on the strength of spin-orbit interaction (through the parameter η) and the beams' separation d. We emphasize that this shift of the resonance is an entirely new result that so far was not anticipated in the literature. We understand the resonance shift as follows. Depending on the direction of electron diffusion (in the positive or negative x direction), the electron spins observed by the first beam acquire a spin rotation in the clockwise or counterclockwise direction (due to the spinorbit interaction) by the moment of time when some of them are tested by the second beam in its own region of space. This spin rotation angle is added or subtracted from the Larmor precession angle causing a positive or negative peak shift.

Fig. 2 represents the spin noise power spectrum obtained as the Fourier transform of Eq. (7) for two values of d. Clearly, the peaks are of Lorentzian shape and $d\eta=4\pi$ peak is shifted by δ with respect to $d\eta=0$ peak. While the half width at half-maximum (HWHM) gives information about the possible spin relaxation channels, the value of the peak shift δ allows finding η , so that the strength of the DP spin relaxation and its role in overall spin relaxation can be determined.

Interestingly, additional effects can be observed if we consider a non-equilibrium situation e.g. by applying a

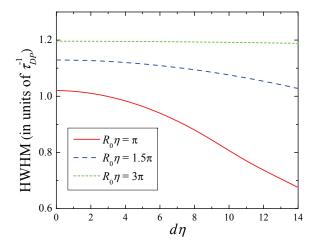


FIG. 4. Half width at half-maximum (HWHM) for the peak in the spin noise power spectrum (Fig. 2) as a function of beams' separation d. These curves were obtained using the parameter values $\gamma = 0.2/\tau_{DP}$ and $\omega_L = 5/\tau_{DP}$.

weak external electric field E along the wire. The diffusion part of the propagator in (6) is then modified to $\frac{1}{\sqrt{4\pi Dt}} \exp\left(-(x-x'-\mu Et)^2/4Dt\right)$, where μ is the electron mobility. By neglecting $\sim E^2$ terms in corresponding R(t) function, the integration over the beam intensities gives us

$$S(\omega; E) \propto \frac{\tau_s^{-1} + \frac{d\mu E}{2R_0^2}}{(\tau_s^{-1} + \frac{d\mu E}{2R_0^2})^2 + (\omega_L + \delta - \eta\mu E - \omega)^2} e^{-\frac{d^2}{4R_0^2}},$$
(10)

According to this result, the effect of the electric field is twofold. First, it renormalizes the effective correlation time τ_s because spins moving along the electric field have a better chance to travel the distance d between the two beams before they relax, than spins moving against the electric field. This effect should be observable as the change of the peak amplitude as a function of the electric field. Perhaps more importantly, the electric field renormalizes the position of the peak maximum: $\delta \to \delta - \eta \mu E$. This shift can be used as an independent probe of the spin orbit coupling η . It emerges because the nonzero average drift velocity μE , via the spin orbit coupling, creates on average an effective transverse Zeeman-like field acting on electron spins with the corresponding Larmor frequency $\approx \eta \mu E$. This effect is not specific for quasi-1D geometry. It should emerge in any material with a linear in momentum spin-orbit coupling, such as the Rashba and the Dresslhaus couplings in 2D electron systems and strained 3D semiconductors. For example, taking parameters from Rashba electron system¹⁷: $\mu = \sigma_{xx}/e$, where $\sigma_{xx} = 1.8 \cdot 10^3 \Omega^{-1} \text{m}^{-1}$ is the conductivity, and $\eta = 1.5 \cdot 10^{-13} \text{eV} \cdot \text{m}$, we find that an electric field $E \sim 10 \text{eV/cm}$ shifts the spin noise power peak by $\sim 100 \mathrm{MHz}$, which should be clearly observable in GaAs.

Finally, we would like to consider δ and HWHM be-

yound the limiting case discussed above. For this purpose, we perform exact calculations of noise power spectrum (based on Eq. (7)) and extract δ and HWHM from these calculations. Figs. 3, 4 present some of our results. We note that while the peak shift scales almost linearly in the range of selected parameter values, the peak width demonstrates a significant variation at the smallest values $R_0\eta$. Consequently, in the latter case the spin relaxation time should be extracted using the Fourier transform of the exact spin noise correlation function.

In conclusion, as the peak shift and width can be easily determined from experimental measurements, we anticipate that the suggested approach of two-beam spin noise spectroscopy will be extremely useful in the area of semiconductors and emergent materials. Possible applications of the two-beam spin noise spectroscopy include the possibility to separate contributions of conduction and localized electrons to the spin noise, to identify the role of different spin relaxation mechanisms contributing to the process of electron spin relaxation in a particular material, and to measure spin orbit coupling anisotropy.

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